Important Tips on Quadratic Equations

for Air Force Group X & Y

gradeup

Important Tips and Tricks on Quadratic equation for Air Force Group X & Group Y

First and foremost, Relation between X and Y is established only when the relationship is defined for all solutions.

1. Linear Equations: In linear equations, both X and Y have only one value. So relation can be established easily.

4X+3Y=18, 7x+5Y= 12

(4X+3Y= 18)× 5, (7X+5Y=12)× 3

20X+15Y=90……..(i)

21X+15Y=36……..(ii)

subtracting equation (i) from equation (2)

we get, X = -54, Y = 78

Hence, Y > X

2.Squares: In this, solutions have both negative and positive value.

X^2=1600 and Y^2=3600

X = ±40 and Y = ±60

+60 is greater than both -40 and +40, but -60 is less than both -40 and +40. So, the answer will be Cannot be determined.

TRICK: Whenever both equations are given in the square form, our ANSWER will be 'Can't be determined.'

3. Squares and Square root case.

X^2=1600 and Y = √3600

We know that square root always gives a positive value. So, Y will have **ONLY +60 NOT -60**.

X = ±40 and Y = +60

+60 is greater than both +40 and -40. Hence Y > X.

4. Cubes Case.

If X^3=1331, Y^3=729

then, X=11 and Y = 9

X is greater than Y, so relation is X > Y.

If X^3= -1331 and Y^3= 729

then, X= -11 and Y = 9

X is greater than Y, so relation is X < Y.

Note: Can you see something common in above example? Common thing is that when $X^3 > Y^3$, relationship is X > Y and when $X^3 < Y^3$, relation is X < Y.

TRICK: When both equations are in cube form. If $X^3 > Y^3$, then X > Y and $X^3 < Y^3$, then X<Y.

5. Square and cube cases.

If X^2=16 and Y^3=64

then X = +4, -4 and Y=4

So, Y = 4 is equal to X = 4 and Y = 4 is greater than X = -4.

So, Y ≥ X

If X^2=25 and Y^3=64

then X = +5, -5 and Y = 4

So, Y = 4 is greater than X = -5 and less than X = +5, So relation Can't be Determined.

Table Method to solve Quadratic Equations Easily

1. Write down the table (given below) before exam starts, in your rough sheet, to use during the exam, Analyse the (+, -) signs in the problem, and refer to the table of signs.
2. Write down the new (solution) signs, and see if a solution is obtained instantly. If not, then go to step 3.
3. Obtain the two possible values for X & Y, from both the equations,
4. Rank the values and get the solution,

STEP 1

Firstly, when you enter the exam hall, you need to write down the following **master table** in your rough sheet instantly (only the signs):-
Let us consider that the equations are $AX^2+BX+C = 0$ and $AY^2+BY+C = 0$

Type of Equation	$AX^2+BX+C = 0$ or $AY^2+BY+C = 0$		Roots in X or Y equation	
	Sign of BX or BY	Sign of C	Sign of bigger root	Sign of smaller root
P	+	+	-	-
Q	-	+	+	+
R	+	-	-	+
S	-	-	+	-

Now we will discuss the cases as mentioned below in the table.

CASE	ROOTS OF X /Y	ROOTS OF X/Y	CONCLUSION
I	**+,+ (Q)**	**+,+ (Q)**	**Easy**
II	**+,+ (Q)**	**+,- (R or S)**	**Will discuss**
III	**+,+ (Q)**	**-,- (P)**	**Left>Right**
IV	**+,- (R or S)**	**-,- (P)**	**Will discuss**
V	**+,- (R or S)**	**+,- (R or S)**	**Cannot be defined**
VI	**-,- (P)**	**-,- (P)**	**Easy**

CASE I: When the result of both equations are Q-type having both roots (+).

(i) If X^2-5X+6 = 0
both roots will be positive i.e. +3 and +2
Y^2-17Y+66 = 0
both roots will be positive i.e. +11 and +6
We can see that both roots of X are less than both roots of Y. So, X < Y.
(ii) If X^2-17X+42
both roots will be positive i.e. +14 and +3.
Y^2-17Y+66 = 0
both roots will be positive i.e. +11 and +6.
Here, +14 > +11
but +14 < +6
also +3 < +11
and +3 < +6
As we can see in the comparison above, there are TWO relations between X and Y which are both > and <. So **relation cannot be defined.**

Note 1: When both equations have BX (-) and C(+), You have to go in detail.

CASE II: When the result of one equation is Q type and another is either R type or S type.

(i) Q type: Y^2-49Y+444, Roots are 37,12
R type: X^2+14X-1887, Roots are -51,37
Now let us compare the values in this table below –

X	RELATION	Y
-51	<	37
-51	<	12
37	=	37
37	>	12

When we compared the values of X and Y in the table above, we found that there are THREE relations between X and Y i.e. =, > and <. So, a relation **cannot be defined**.
(ii) Q type: X^2-5X+6 = 0, Roots are 3,2
R type: Y^2-Y-6 = 0, Roots are 3,-2
Now let us compare the values in this table below –

X	RELATION	Y
3	=	3
3	>	-2
2	<	3
2	>	-2

When we compared the values of X and Y in the table above, we found that there are TWO relations between X and Y i.e. >, =. **So relation CANNOT BE DEFINED.**

CASE III: When one equation is P-type having both roots (-) another Q type having both roots (+).

(i) P type: $X^2+5X+6=0$, Roots are -3, -2

Q type: $Y^2-7Y+12=0$, Roots are 4,3

Comparing the values in the below table –

X	RELATION	Y
-3	<	3
-3	<	2
-2	<	3
-2	<	2

On comparing, we saw that So roots of Y equation are greater than roots of X.

Note: In this case, roots of the equation Q type will always be greater than P-type.

CASE IV: When the result of one equation is P-type having both roots negative and another is either R type or S type having one root (-) and another one (+)

(i) P type: X^2+5X+6, Roots are -3,-2

R type: X^2-X-6, roots are 3,-2

Comparing the values in the below table –

X	RELATION	Y
-3	<	3
-3	<	-2
-2	<	3
-2	=	-2

On comparison of X and Y values, There are THREE relations between X and Y i.e. =, > and <. So relation **cannot be defined**.

(ii) P type: X^2+5X+6, Roots are -3, -2

R type: X^2-X-6, Roots are 3,-2

Now let us compare the values in this table below –

X	RELATION	Y
-3	<	3
-3	<	-2
-2	<	3
-2	=	-2

When we compared the values of X and Y in the table above, we found that there are TWO relations between X and Y i.e. <, =. **So, the relation is X ≤ Y.**

Case V: When the result of both equations are either R type or S type or one equation is R type and another is S type having one root (-) and another root (+).

(i) If $X^2+X-6 = 0$

Roots are -3 and +2.

$Y^2+5Y-66 = 0$

Roots are -11 and +6.

Comparing the values in the below table –

X	RELATION	Y
-3	>	-11
-3	<	+6
+2	>	-11
+2	<	+6

On comparison of X and Y values, there are two relations between X and Y i.e. both > and <. So, the relation **cannot be defined**.

(ii) If $X^2+11X-42$

Roots are -14 and +3

$Y^2+5Y-66 = 0$

Roots are -11 and +6

Comparing the values in the below table –

X	RELATION	Y
-14	<	-11
-14	<	+6
+3	>	-11
+3	<	+6

When we compared the values of X and Y in the table above, we saw that there are two relations between X and Y i.e. both > and <. So, the relation **cannot be defined**.

(iii) If $X^2-X-6 = 0$

Roots are +3 and -2

$Y^2-5Y-66 = 0$

Roots are +11 and -6.

Let us compare the values of X and Y in below table –

X	RELATION	Y
+3	<	+11
+3	>	-6
-2	<	+11
-2	>	- 6

As the table shows, there are two relations between X and Y i.e. both > and <. So, relation **cannot be defined**.

(iv) If X^2-11X-42

Roots are +14 and -3

Y^2-5Y-66 = 0

Roots are +11 and -6

We are comparing these roots of X and Y in the table below –

X	RELATION	Y
+14	>	+11
+14	>	-6
-3	<	+11
-3	>	-6

On comparing, there are two relation between X and Y i.e. both > and <. So relation **cannot be defined**.

Note 4: In this case, the answer will always be CANNOT BE DEFINED.

CONCLUSION: Whenever in a question, the sign of C is negative (-) in both X and Y equation, then answer will always be CANNOT BE DEFINED.

Case VI: When the result of both equations are P-type having both roots (-).

(i) If X^2+5X+6 = 0

Both roots will be negative i.e. -3 and -2

Y^2+17Y+66 = 0

Both roots will be negative i.e. -11 and -6

We can see that both roots of X are greater than both roots of Y. So, **X > Y.**

(ii) If X^2+17X+42

both roots will be negative i.e. -14 and -3.

Y^2+17Y+66 = 0

both roots will be negative i.e. -11 and -6.

On comparing these values of X and Y in the table below –

X	RELATION	Y
-14	<	-11
-14	<	-6
-3	>	-11
-3	>	-6

We found that there are two relations between X and Y i.e. both > and <. So relation **cannot be defined**.

Note 6: When both equations have BX (-) and C(+), You have to go in detail.

Other Examples:

$2l^4 - 36l^2 + 162 = 0$ and $3m^4 - 75m^2 + 432 = 0$

Solution: Basically this is not a quadratic equation because the maximum power of variable is 4.

But if you suppose l^2 is X and m^2 is Y, then equations will be $2X^2-36X+162=0$ and $3Y^2-75Y+432=0$

Now the converted equations are Q type having all roots positive.

$X = l^2$ = positive roots, hence **l** will have 2 negative roots and 2 positive roots.

$Y = m^2$ = positive roots, hence **m** will also have 2 negative roots and 2 positive roots.

So the relation between **l** and **m** cannot be defined.

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